Feedback Control of Transitional Channel Flow using Balanced Proper Orthogonal Decomposition

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Reduced-order models of linearized channel flow are used to develop closed-loop controllers which are then applied to full nonlinear direct numerical simulations of the flow. The models are computed using Balanced Proper Orthogonal Decomposition (BPOD), which has been shown to yield models which capture well the input/output behavior of linear perturbations in transitional flow. We investigate the control of disturbances using either localized body force or wall blowing and suction as actuation. We introduce a method of treating the inhomogeneous boundary conditions in computing the simulation snapshots for BPOD, based on regularizing the inhomogeneous particular solution which arises in the standard lifting approach for wall blowing and suction. For localized body force actuation, the closed-loop controller is able to prevent the transition to turbulence for initial perturbation amplitudes which in the uncontrolled case transition due to linear non-normal growth. Energy growth reduction for an optimal perturbation at a low Reynolds number using a model-based controller and sinusoidal wall blowing and suction in the streamwise direction as actuation is also demonstrated.

I. Introduction

Active flow control is of interest in many practical applications, and extensive research has been done recently in the field. With the development of advanced computer simulation methods, control theory methods1–3 and experimental techniques,4 combined with efforts towards better communication among these fields,5 advances have been made towards successful control of shear flows. A particularly useful approach in this field is the development of reduced-order models, both for practically feasible control design and for learning more about the underlying flow physics.

In this paper we present a continuation of previous work,6 where it was shown that low-order models for a localized body-force actuator in transitional channel flow obtained using Balanced Proper Orthogonal Decomposition (BPOD) capture the input-output behavior very well. In particular, these models performed much better than standard POD, which is the commonly used method for model reduction in fluid mechanics. Here we develop feedback controllers based on these models and assess their performance when applied to the full nonlinear system. We also investigate wall blowing and suction as a more practical actuation mechanism.

II. Model Reduction via BPOD

Balanced truncation is a standard method for model reduction, first introduced by Moore,7 which is applicable primarily to linear, stable, time-invariant systems, although there are extensions for nonlinear systems,8 unstable systems9 or periodic systems.10 The main difficulty in using balanced truncation for large systems such as those typically encountered in fluid problems is computational expense — traditional approaches are not feasible. Recently, a snapshot-based method for balanced truncation of large linear systems was introduced.1 For systems with large numbers of outputs (e.g., if the output is the entire state), the snapshot-based method requires a large number of adjoint simulations, which still makes the problem computationally intractable. A procedure we call Balanced POD (BPOD)3 uses an output projection to reduce the number of necessary adjoint simulations, and has been shown to be a computationally feasible
approximation to balanced truncation for examples of two- and three-dimensional perturbations to laminar channel flow,\textsuperscript{1,6,11} as well as for two-dimensional flow past a flat plate at an angle of attack.\textsuperscript{12,13} Recently a similar procedure was applied to control of two-dimensional channel flow.\textsuperscript{14}

Obtaining reduced-order models via BPOD from a large simulation with output projection is described in detail in the references,\textsuperscript{6} and here we outline the main steps. Starting from a system in state-space form given by
\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx,
\end{align*}
\]
we perform the following steps:

1. Compute the state impulse response for each input, given by \( x(t) = e^{At}B \).
2. Compute the POD of the state impulse response and stack the modes into a matrix \( \Theta \).
3. Select the number \( s \) of POD modes \( \theta_j \) for output projection. This is equivalent to choosing how much error we tolerate between the outputs of the full system and the output-projected system.
4. Compute the required adjoint simulations \( z(t) = e^{A^+t}C^+ \) where \( A^+ \) is the adjoint operator to \( A \), and the columns of \( C^+ \) are the POD modes \( \theta_j \) where \( j = 1, \ldots, s \).
5. Assemble the snapshots from steps 1 and 4 into the matrices \( X \) and \( Y \) respectively, construct the matrix \( S = Y^TX \) (where \( S_{mn} = \langle x^n, z^m \rangle \)) and compute the singular value decomposition \( S = U\Sigma V^T \).
6. Compute the balancing modes \( \phi_j \) and \( \psi_j \), which are the columns of the matrices \( \Phi = XV\Sigma^{-1/2} \) and \( \Psi = YU\Sigma^{-1/2} \) respectively.

Using these modes, the reduced-order model becomes
\[
\begin{align*}
\dot{a} &= \Psi^+A\Phi a + \Psi^+Bu \\
y &= \Theta^T_s C\Phi a
\end{align*}
\]

Depending on how the adjoint operator is derived, appropriate inner product weights may need to be included in computing the matrix \( S \).\textsuperscript{6}

### III. Linearized Channel Flow

The governing equations for the evolution of a linear perturbation to laminar channel flow can be written in standard state-space form:
\[
\begin{align*}
\dot{x} &= Ax + Bu_1 + Fu_2 \\
y &= Cx
\end{align*}
\]
where \( u_1 \) represents an actuator (or vector of actuators) and \( u_2 \) represents a vector of disturbances, with matrices \( B \) and \( F \) representing the corresponding effects on the dynamics. Here, the state vector \( x \) is defined as \( x = [v \ \eta]^T \), where \( v \) and \( \eta \) are the wall-normal velocity and wall-normal vorticity, respectively. Here, the \( A \) matrix is a discretization of the linearized Navier-Stokes equation, which we give below in operator form:
\[
A = \begin{bmatrix} -\Delta & 0 \\ 0 & I \end{bmatrix}^{-1} \begin{bmatrix} L_{OS} & 0 \\ -U'\partial_x & L_{SQ} \end{bmatrix}
\]
where
\[
L_{OS} = U\partial_x\Delta - U''\partial_x - \frac{1}{Re}\Delta^2
\]
\[
L_{SQ} = -U\partial_x + \frac{1}{Re}\Delta
\]
are the Orr-Sommerfeld and Squire operators, respectively, with appropriate boundary conditions. Here, \( Re = U_c\delta/\nu \) is the Reynolds number, where \( \nu \) is the kinematic viscosity, \( \delta \) is the half-width of the channel,
and $\Delta = \partial_x^2 + \partial_y^2 + \partial_z^2$ is the Laplacian. $U_c$ is a characteristic velocity, which for linearized channel flow is the centerline velocity of the laminar profile $U(y)$. The prime indicates differentiation with respect to $y$. This system has been widely investigated\textsuperscript{15–18} and the transient non-normal growth of exponentially stable perturbations has been identified as a likely cause for the so-called bypass transition which has been observed in shear flows. We are interested in the whole flow field of the perturbation, whose non-normal energy growth should be suppressed in order to delay or prevent transition. The output matrix $C$ in our case is therefore the identity matrix (i.e., the output is the whole state). When a single measurement is of interest as the system output, or the full flow field is reconstructed using an estimator based on a single or a small number of measurements, the system can have a single output or just a few outputs and an output projection is not necessary.

The adjoint operator derived with respect to a numerically convenient inner product\textsuperscript{6} is given by

$$A^+ = \begin{bmatrix} -\Delta & 0 \\ 0 & I \end{bmatrix}^{-1} \begin{bmatrix} L^*_{OS} & U'\partial_z \\ 0 & L^*_{SQ} \end{bmatrix} \quad (5)$$

where

$$L^*_{OS} = -U\partial_x \Delta - 2U'\partial_x \partial_y - \frac{1}{Re} \Delta^2$$
$$L^*_{SQ} = U\partial_x + \frac{1}{Re} \Delta.$$

### IV. Control Design

In this section we discuss the design of reduced-order controllers for very large systems such as transitional channel flow. Two possible design strategies are illustrated in Figure 1. Controllers for the full plant have been designed for channel flow by Högberg et al.\textsuperscript{19} Although it is computationally expensive, such design is feasible in the case of channel flow, since the problem is decoupled into streamwise and spanwise wavenumbers. For other large systems, and in particular spatially developing flows, or flows with more complicated boundary conditions, this approach may not be feasible, and a reduced-order controller needs to be designed based on a reduced-order model of the plant. For instance, this method has been applied by Cortelezzi and Speyer\textsuperscript{20} for two-dimensional channel flow, where a controller was designed and then reduced at each streamwise wavenumber. In this work we investigate controller design for reduced-order models of three-dimensional channel flow without decoupling the flow into components at each wavenumber, while a comparison to the strategy of designing a full order controller first and then reducing its order is a subject of our future work.

![Figure 1. Two possible strategies for designing feedback control for large systems. Both strategies are computationally feasible in channel flow.](image)

In this investigation, a standard linear quadratic regulator (LQR) was designed. LQR design is described in detail standard references on control theory.\textsuperscript{21} The feedback gain for the system (1) is given by $u = -Kx$, where the gain matrix $K$ is computed so that it minimizes the objective function

$$J = \int_0^\infty (x^T Q x + u^T R u) \, dt. \quad (6)$$
Given the state-space system and the weight matrices $Q$ and $R$, the gain $K = R^{-1}B^T P$ is computed from the solution of the Riccati equation

$$A^T P + PA - PB R^{-1}B^T P + Q = 0. \quad (7)$$

Computation of the control gain for the full system (1) is clearly not feasible for three-dimensional fields in transitional channel flow, as the required matrices cannot even be stored in memory for a large system such as a linearized DNS. Instead, the gains are computed for the BPOD low-order models (which can be done in MATLAB with negligible computational expense) and then applied to simulations of the full non-linear system. The weight $Q$ on the state is chosen such that $x^T Q x$ is the total kinetic energy in the perturbations, while the actuator penalty $R$, which in this single-input case is just a scalar, is used as a tuning parameter. The control design in this work assumes that the full state is available (i.e., an estimator was not used). The linear controllers were tested on a full nonlinear DNS of the Navier-Stokes equations based on the method by Kim et al$^{22}$ with imposed constant mass flux. A linearized version of the same code was used for the linear simulations from which the snapshots for the models were obtained.

### V. Wall Blowing and Suction as Actuation

#### A. Lifting approach

While the implementation of wall blowing and suction into a DNS simulation is straightforward, the computation of the corresponding reduced-order models for control design requires some care. For model reduction it is of particular importance to work with modes which have homogeneous boundary conditions. A standard formulation of the corresponding reduced-order models for control design requires some care. For model reduction, Homberg et al.$^{19}$ allows the treatment of inhomogeneous boundary conditions when designing the control gains. The full flow field is given by $x = x_h + x_p$, where $x_h$ satisfies homogeneous boundary conditions ($u = v = v_y = 0$ at walls), and $x_p$ is a particular solution satisfying the desired wall-blowing boundary condition. In our reduced-order models, we expand $x_h$ in terms of modes that satisfy the homogeneous boundary conditions, and write $x_p = Z \phi$, where $\phi(t)$ is the amplitude of the wall blowing/suction, and $Z$ is chosen to be a steady-state solution of the linearized equations with boundary condition $v = 1$ at the wall, as in Högberg et al.$^{19}$ For a single wavenumber pair this solution can easily be computed using a spectral collocation method, while for a localized actuator on the channel wall it can also be computed by starting the simulation impulsively with the wall blowing and suction actuator as an initial condition and letting the solution reach steady state.

After augmenting the full state with the values of $\phi$, and neglecting the disturbance term in (3), the equations become

$$\begin{bmatrix} \dot{x}_h \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} A & AZ \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_h \\ \phi \end{bmatrix} + \begin{bmatrix} -Z \\ I \end{bmatrix} \dot{\phi},$$

where the input is now $u_1 = \dot{\phi}$, where the dot indicates a time derivative. Some of the alternative approaches to lifting for treating inhomogeneous boundary conditions are actuation modes$^{23}$ or using a weak formulation.$^{24}$ In the former approach, the actuation is not captured by the reduced-order models in a systematic fashion, rather it is treated separately from the system dynamics, while in the latter approach the modes have inhomogeneous boundary conditions, which can cause difficulties in applying feedback control.$^{24}$

#### B. Regularization of forcing term

In order for POD or BPOD modes to satisfy homogeneous boundary conditions, it is necessary for all of the snapshots used to compute the modes to have homogeneous boundary conditions as well (this is obvious, since in both cases, the modes are linear combinations of the snapshots). The particular solution given by the matrix $Z$ described above is nonzero at the boundaries, so an impulse response of (8) would have a nonzero boundary condition at $t = 0^+$. Thus, using the snapshots from this impulse response (or equivalently, with the zero-input solution using $-Z$ as the initial condition for $x_h$) would result in modes that do not satisfy homogeneous boundary conditions, or that have a discontinuity at the wall. In order to avoid this problem, we regularize the initial condition $Z$ by spatial filtering. In particular, we use an inverse Laplacian filter, where the smoothed version $u_s$ of a function $u$ is given by

$$ (1 - \gamma D^2) u_s = u \quad (9) $$
where $\gamma$ is a tunable parameter. Here $D$ indicates differentiation in the wall-normal direction. The procedure of obtaining the initial condition for 3-D simulations is to compute the regularization in wall-normal direction at each $x, z$ wavenumber pair. A comparison of the unfiltered particular solution in the wall-normal direction for a given combination of streamwise and spanwise wavenumbers and Reynolds number, and the regularized value is shown for three different values of $\gamma$ in Figure 2. A regularized localized actuator with zero-net-mass-flux wall blowing/suction at $Re = 2000$ is shown in Figure 3. We note here that the linearized channel flow is a fourth-order system, and that the regularized particular solution should satisfy the clamped boundary conditions. It was found in our simulations that regularizing with Dirichlet boundary conditions only was sufficient, as the Neumann boundary condition on the velocity is quickly imposed by the numerical simulation.

![Figure 2](image2.png)

**Figure 2.** The real part of the particular solution $Z$ for the wall-normal velocity at the wavenumber pair $\alpha = 1, \beta = 0$ and $Re = 2000$ and regularized solutions for three different values of the parameter $\gamma$.

![Figure 3](image3.png)

**Figure 3.** a) Localized zero-net-mass-flux actuator on channel bottom wall, b) The corresponding particular solution, $v$ component. c) Regularized $v$ component of the particular solution.

An intuitive justification for this filtering is as follows: physically, an impulse response of (8) corresponds to a step change in wall blowing, and it is this step change that leads to the spatial discontinuity in the solution. In any real system, this change in wall blowing could not be instantaneous, so this discontinuity would not actually arise in a physical system. Even if the step change were instantaneous, viscosity would quickly smooth any discontinuities. The filter (9) may be viewed as adding a small amount of numerical viscosity to smooth this discontinuity as soon as it is introduced in the impulse response simulations.

C. **Controller Design**

The control gains for wall blowing and suction are designed as described in Section IV, although the energy weight matrix in this case is different, since the state is augmented by the particular solution $x_p = Z\phi$. If we write the the low-order approximation of the homogeneous part of full state of the system as $x_h = \Phi_r \alpha$, where the columns of $\Phi_r$ are the first $r$ balancing modes, from the expression for the energy norm of $x_r$, calculated.
\[ \|x_r\|^2 = (x_h + x_p)^TQ(x_h + x_p) \]
we see that the state weight matrix for a reduced-order model of rank \( r \) is written as follows:
\[
\hat{Q} = \begin{bmatrix}
\Phi^T r Q \Phi_r & \Phi^T r Q Z \\
Z^T r Q \Phi_r & Z^T r Q Z
\end{bmatrix},
\] (10)
where \( Q \) is the energy weight matrix for the full system. The last two rows and columns of the matrix \( \hat{Q} \) take into account the effect of the actuation on the walls. Thus, the feedback control gains in this case are obtained by solving a \((r + 2) \times (r + 2)\) Riccati equation:
\[
\hat{A}^T P + P \hat{A} - P \hat{B} R^{-1} \hat{B}^T P + \hat{Q} = 0,
\] (11)
where
\[
\hat{A} = \begin{bmatrix}
\Psi^+_r A \Phi_r & 0 \\
0 & 0
\end{bmatrix}, \quad \hat{B} = -\Psi^T B
\] (12)
The gains are now obtained as \( \hat{K} = R^{-1} \hat{B}^T P \). As described by Högberg et al., the contribution of the homogeneous part of the flow to the control gains at the walls needs to be subtracted from the computed values. We note here that the control input is the time derivative of the wall blowing and suction, which should be integrated in time. The time advancement of the control input is computed at each step in the DNS simulations, using the same numerical scheme as the one used in advancing the solution in time.

VI. Results

For a linear perturbation of a given structure, the lowest initial perturbation energy (dependent only on the amplitude), which will result in the perturbation transitioning to turbulence, is defined as the transition threshold. The increase in transition thresholds as a result of applying feedback control was studied by Högberg et al. for a class of linear perturbations, and we perform a similar investigation here for controllers based on BPOD models, using both localized body forces and wall blowing and suction as actuation mechanisms.

A. Localized Body Force Control

We first consider actuation with a localized body force in the center of the channel. The perturbation in this case is also a localized body force in the channel center for \( Re = 2000 \), as described in Ilak and Rowley.

When using output projection, the states of the reduced-order model are mode coefficients. In order to compute feedback we need to know these coefficients, which are computed by projecting the full simulation at a given time onto the basis of balancing modes used to form the reduced-order model. Thus, even though a low-order model is used, our control strategy is typical of what is known as full-information control or full-state feedback in the control community. Although this case is not literally possible in practice, full-information control is an important first step in studying how the flow is altered by introducing feedback actuation. Figure 4 illustrates the closed-loop control setup for this case.

![Figure 4](image-url)

Figure 4. A schematic representation of the closed-loop control using localized body force actuation. The coefficients of the modes \( \hat{a} \) are obtained by projection of the full field onto the balancing modes, and the gains \( \hat{K} \) are obtained from solving a rank three Riccati equation.

The controller used here was designed using the reduced-order model of rank three, which has been shown to capture well the open-loop dynamics of the linearized system, and in particular the energy growth, which
is dominated by streamwise-constant modes of low spatial wavenumber in the spanwise direction. Figure 5 (a) shows the performance of the model in capturing the uncontrolled perturbation energy growth and Figure 5 (b) shows the first three balancing modes. It is clearly seen that the dynamics of the linearized perturbation is dominated by streamwise-constant modes (corresponding to streamwise wavenumber $\alpha = 0$) with fairly low spanwise wavenumbers $\beta$.

Figure 5. (a) The energy growth of the uncontrolled linearized perturbation for the full simulation and for a 3-mode BPOD model. The wall-normal velocity field of the perturbation is shown in the inset, with the white isosurface indicating positive value and the dark isosurface negative value. (b) The first three BPOD modes.

Figure 6 (a) shows the uncontrolled evolution of the perturbation energy for four different initial energy values, with two of the simulations transitioning to turbulence (the perturbation energy is defined as the energy of the flow field with the corresponding laminar profile subtracted). The transition threshold for the uncontrolled case is approximately at $r = 1.614 \times 10^{-4}$, where $r$ is defined as $E_0/E_{lam}$, $E_0$ being the initial perturbation energy density (energy per unit volume) and $E_{lam} = 0.2667$ being the energy density of the laminar flow. Figure 6 (b) shows a logarithmic plot of the energy in the linear, uncontrolled and two different successful controlled cases. The closed-loop controllers have been found to work well when the authority of the controller is relatively high.

The mechanism of transition for the localized perturbation is illustrated in Figure 7, which shows the spatial Fourier transform of the wall-normal velocity for the $x,z$-plane of the channel at $y = 0$ at $t = 40$ for four cases - linear and nonlinear uncontrolled evolution, and two controlled cases (with $R = 1000$ and with a more aggressive controller with $R = 0.1$). In the linearized case (a), the spatial wavenumbers are confined only to the set which is contained in the initial perturbation, and no new wavenumbers are generated (the Gaussian-like perturbation does contain all the wavenumbers which are present in the computational domain, but the amplitudes of the higher modes are negligible) and the perturbation decays at subcritical Reynolds numbers. In the nonlinear case (b), the nonlinear interaction among different modes gives rise to higher harmonics, in particular those with higher spanwise wavenumbers $\beta$ and zero streamwise wavenumber (this phenomenon has been described by Henningson et al\textsuperscript{26} as the $\beta$-cascade). If the amplitude of these modes is high enough, they alone can lead to transition, even if the growth of lower wavenumbers is suppressed by the control.

As we have seen, the BPOD models capture well the dynamics of the linearized perturbation, which has significant contributions only at low spatial wavenumbers, and the BPOD modes do not contain high-$\beta$ wavenumbers. Therefore, the closed-loop controller is unable to suppress the growth of these higher $\beta$ modes. Figure 7 (c) and (d) show two different controller cases for the same initial amplitude of the perturbation: one where the low spatial wavenumbers are successfully controlled (c), but their initial nonlinear interaction still gives rise to higher wavenumbers at sufficient amplitudes for transition, and the other for a more 'aggressive' controller (d) which immediately decreases the amplitude of the low spatial wavenumbers so much that the
Figure 6. Left: uncontrolled perturbation energy evolution for different values of the ratio $r$ for the localized perturbation, indicating the approximate threshold value. Right: energy growth for linear and non-linear uncontrolled evolution of the perturbation at initial energy density ratio $r = 3.323 \times 10^{-4}$, and for controlled cases with the same $r$ and LQR actuator weights $R = 0.1$ and $R = 0.01$. The inset shows the initial energy growth on a linear scale indicating the fast initial drop in the perturbation energy for the successfully controlled cases.

Figure 7. Spatial Fourier transform for the $x,z$–plane at $y = 0$ at $t = 40$. Linear evolution (a), non-linear evolution (b), closed-loop non-linear evolution for controller with actuation penalty $R = 1000$ (c) and $R = 0.1$ (d). Note the difference in the magnitude of the wavenumbers between the third and fourth figures.
arising higher harmonics are of such low amplitude that they decay after some linear non-normal growth, and the flow subsequently re-laminarizes. For the controlled cases with $R = 0.01$ (the most ‘aggressive’ control design attempted) it was found that the transition threshold ($r = 2.843 \times 10^{-3}$) is about 17 times higher that in the uncontrolled case ($r = 1.614 \times 10^{-4}$).

It is important to keep in mind the simplicity of the design of the controller used here. The controller used by Högberg et al\textsuperscript{19} was composed of feedback gains at each wavenumber pair $\alpha, \beta$ present in the simulation and was therefore successful at attenuating perturbation growth at all wavenumbers. This type of controller, however, is complex to compute as it requires the computation of three-dimensional convolution kernels for the whole field, while the closed-loop controller designed here is a simple third-order linear system. The use of controllers based on higher-order models did not yield an improvement over the performance presented here, since the high spatial wavenumbers, which are insignificant in the linear case but are sufficient to cause transition above a certain initial amplitude in the nonlinear case, are not at all captured by the balancing modes. One alternative approach which could be used to overcome this drawback is the use of an empirical nonlinear balancing technique, such as the one suggested by Lall et al.\textsuperscript{8}

B. Wall Blowing and Suction

Controllers were designed using wall blowing and suction at the top channel wall only, at a single wavenumber pair $\alpha = 1, \beta = 0$, with $Re = 2000$. This form of sinusoidal wall blowing and suction in the streamwise direction has been used in several studies.\textsuperscript{3, 27, 28} We first attempt to suppress the growth of the optimal perturbation (resulting in the largest possible energy growth) at the same wavenumber pair via blowing/suction at the upper channel wall. We first compute feedback for the full system, and we apply both exact balanced truncation and BPOD with the regularization of the particular solution as described in Section V. The initial condition shown in Figure 2 with $\gamma = 0.0013$ was chosen for computing the BPOD models.

We note that the exact balanced truncation for this 1-D linear problem is computed for comparison with BPOD, and would not be computationally feasible for a three-dimensional localized actuator. The standard spectral collocation method we use imposes homogeneous boundary conditions by neglecting the boundary points at $y = \pm 1$, although the particular solution $Z$ and therefore the resulting modes have a discontinuity at the upper wall. Figure 8 shows the suppression in perturbation energy growth using a closed-loop controller designed from a 10-mode BPOD model and the corresponding control input (the wall-normal velocity on the upper wall $\phi(t)$) and its derivative. The rank of output projection for the BPOD model is 4, and the actuator penalty is $R = 10$ for both the full system and model-based controllers. We notice that the performance of the BPOD model with the regularized initial condition is inferior to that of the exact balanced truncation, although the suppression of energy growth is still significant. On the other hand, BPOD computed without regularizing the actuator yields the same performance as the exact balanced truncation result shown, indicating that the regularization of the initial condition can impact the model performance.

The computation of BPOD models which incorporate localized actuators such as the one shown in Figure 3 and verification that this type of control using BPOD models is successful in DNS simulations is a subject of our present efforts.
Figure 8. a) The suppression of energy growth of the optimal perturbation at $\alpha = 1, \beta = 0$, with $Re = 2000$. b) The amplitude of wall blowing and suction at the lower wall.

VII. Conclusions and Further Work

The transition threshold of a localized perturbation has been increased by an order of magnitude using closed-loop control based on a BPOD model of rank three with localized body force actuation in the center of the channel. While these results are encouraging, the inability of linear models to capture the inherently nonlinear transition mechanism requires controllers of very high authority, which are only successful if high spanwise wavenumbers are not present or have little contribution to the perturbations we are trying to suppress. Some of our current efforts are directed towards an extension of the method of Lall et al\cite{8} for very large nonlinear systems, which may enable the design of more successful controllers.

We have also computed BPOD models for actuation via wall blowing and suction and we have demonstrated that the energy growth of a perturbation can be reduced using a simple controller based on these models. A detailed investigation of the performance of this type of actuation with feedback designed from BPOD models in three-dimensional nonlinear simulations is the main direction of our current efforts and will be reported elsewhere.

The results presented here assume full-state feedback: that is, data from the direct numerical simulation is projected onto the balancing modes at each time step in order to compute the control input. Such full information would certainly not be attainable in practical applications. Wall shear stress measurements have been used in several recent works.\cite{14,19} Another subject of our current work is the use of estimators in order to recover state information from a small number of measurements of wall shear stress and/or pressure and use that information for computing the feedback efficiently in real-time via the low-order models.

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