Lift Enhancement for Low-Aspect-Ratio Wings with Periodic Excitation

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In an effort to enhance lift on low-aspect-ratio rectangular flat-plate wings in low-Reynolds-number poststall flows, periodic injection of momentum is considered along the trailing edge in this numerical study. The purpose of actuation is not to reattach the flow but to change the dynamics of the wake vortices such that the resulting lift force is increased. Periodic forcing is observed to be effective in increasing lift for various aspect ratios and angles of attack, achieving a similar lift enhancement attained by steady forcing with less momentum input. Through the investigation on the influence of the actuation frequency, it is also found that there exists a frequency at which the flow locks on to a time-periodic high-lift state.

Nomenclature

AR = aspect ratio
b = span
C_D = drag coefficient
C_L = lift coefficient
C_m = steady momentum coefficient
\langle C_m \rangle = oscillatory momentum coefficient
c = chord
F, F_y = drag and lift forces
F^* = nondimensional actuation frequency (≡ fL/U∞)
f = actuation frequency
f_{act} = actuator force
f_{bdry} = boundary force
f_n = natural shedding frequency
L = characteristic length of separation
p = pressure
Q = Q criterion
Re = Reynolds number (≡ U∞c/ν)
St = Strouhal number (≡ f_n c sin α/U∞)
s = material coordinate
t = time
U∞ = freestream velocity
u = velocity vector
u_{act} = actuator velocity
x = streamwise, vertical, and spanwise coordinates, (x, y, z)
(x_0, y_0) = actuator position
α = angle of attack

Γ(·) = Heaviside step function
Δt = time step
Δx = grid resolution
δ(·) = regularized delta function
ξ = Lagrangian coordinate
ν = kinematic viscosity
ρ = density
σ = actuator slot width
ϕ = phase
ω = vorticity vector

Introduction

As MICRO air vehicles (MAVs) become smaller in size, the Reynolds numbers at which they operate have approached the range where birds and insects fly [1–3]. For example, Black Widow (AeroVironment, Inc. [4]) and Microbat (California Institute of Technology [5]) operate around Reynolds numbers of 140,000 and 20,000, respectively, while hummingbirds fly at around 20,000. At these low Reynolds numbers, the key feature of the flow is the unsteadiness that is generated by the flying body as well as what is inherently in the flow.

Biofiers are known to exploit the unsteady aerodynamics for enhanced flight performance. For example, birds flap their wings during takeoff and flies are observed to achieve increase in lift from the formation of the leading-edge vortices [6–8]. Unsteadiness in the flow also arises from the perturbation in the flight environment, such as wind gusts, obstacles, or other flying objects in the vicinity [3,9]. Thus, understanding the unsteady nature of the flows over low-aspect-ratio wings is an important problem. In fact, Pines and Bohorquez [10] list it as the first area that requires advancement for future development of MAVs. Relevant investigations have been performed on unsteady separated flow around wings undergoing translation [11,12], pitching/plunging [13–15], and dynamic stall [16,17].

With unsteady flows around the wings, flow control is an attractive option for stable operations of MAVs. Flow control is also desirable as development of MAVs is considered for agile maneuvers, such as those realized by insects. Loitering, hovering, and perching are some of the desirable maneuvers that MAVs could perform that require new approaches in control techniques as nonlinearity in the flowfield would be significant. Thus, novel lift-enhancement approaches are explored in this work, particularly for fixed-wing MAVs.
In the past, a number of flow control studies have been performed. Seifert et al. [18], Greenblatt et al. [19], and Amitay and Glezer [20] have studied the use of periodic excitation for delaying airfoil stall. See Greenblatt and Wygnanski [21] and Seifert et al. [22] for reviews of active separation control using periodic excitation. Circulation control that uses the Coanda effect to increase spanwise circulation for lift enhancement have also been examined and is reported with a comprehensive coverage by Joslin and Jones [23]. Addition of microelectromechanical tabs, called microtabs, near the trailing edge has been considered as well for improving lift [24]. The flow control mechanisms in these past studies have mostly relied on two-dimensionality and have not been reported in the possible aerodynamic benefit from the large-scale three-dimensional effects.

We consider the use of three-dimensional wake vortices and the corresponding vortical forces to enhance aerodynamic performance of low-aspect-ratio wings at poststall angles of attack. Based on our previous study of three-dimensional unsteady vortex dynamics around purely translating low-aspect-ratio wings [12], we have recently considered the application of steady blowing for flow control [25] (also see [26]), an erratum correcting an editorial error). Unlike most other investigations, the objective was not to reattach the flow or delay separation. The goal of steady actuation was to enhance lift at poststall angles of attack by modifying the dynamics of the wake vortices: namely, the leading edge, trailing edge, and tip vortices. As a matter of fact, steady blowing was found to be quite effective in increasing lift (by more than double in many cases) based on a three-dimensional argument for various aspect-ratio wings at poststall angles of attack.

In a continuing effort to use forces generated by the vortices, here we consider the application of a zero-net-mass-flux (ZNMF) actuator, which would require less weight and could be embedded in an MAV more easily than a system necessary for steady blowing. In the current numerical investigation, we mimic the ZNMF actuator by a coarse model of oscillatory momentum injection (external force). The effect of periodic excitation on the wake vortices and the corresponding lift are examined. Comparisons are made with our previous steady flow control study.

The current investigation focuses on \( Re = 300 \), which allows the study to highlight the vortex dynamics in the laminar regime, absent of any turbulence. For higher Reynolds number at which MAVs operate, it is expected that the actuation setup presented below could also affect the large-scale vortical structures behind the aircraft in a similar manner, since the lift-enhancement mechanism to be considered is based on inertial arguments.

### Numerical Approach

#### Spatial and Temporal Discretization

In the current study, incompressible viscous flow around a low-aspect-ratio rectangular wing is simulated with the immersed boundary projection method [27]. The Navier–Stokes equations representing the continuous version of the method are

\[
\frac{D\mathbf{u}}{Dt} = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{u} + \int f_{\text{davy}}(\xi) \delta(\xi - \mathbf{x}) \, ds + \mathbf{f}_{\text{act}}
\]

\[\nabla \cdot \mathbf{u} = 0\]

\[u(\xi) = \int u(x) \delta(x - \xi) \, dx = 0\]

where \( \mathbf{u} \) and \( p \) are the velocity and pressure fields. The boundary force \( f_{\text{davy}} \) is added along the immersed boundary to counteract the oncoming flow to enforce the no-slip boundary condition. A set of Lagrangian points \( \{\xi(s)\} \) with material coordinate \( s \) defines the immersed surface, or the rectangular low-aspect-ratio wing in this study. Also added to the right side of the momentum equation is the actuator force \( \mathbf{f}_{\text{act}} \), which is introduced in the subsection below. The above system of equations is spatially discretized with a second-order staggered-grid finite volume formulation and time-integrated with Crank–Nicholson and second-order Adams–Bashforth schemes for the viscous and convective terms, respectively.

Simulations are performed in a box with a dimension of \([-4.6, 6.1] \times [-5.5, 6.1] \times [-6.6, 6.1] \) in the \( x \) (streamwise), \( y \) (vertical), and \( z \) (spanwise) directions, where the spatial variables are non-dimensionalized by the chord \( c \) of the wing. Grid stretching is applied away from the wing and uniform flow of \( (U_{\infty}, 0, 0) \) is applied at the computational boundary except for at the outflow boundary, where a convective boundary condition \( \partial u / \partial x + U_{\infty} \partial u / \partial x = 0 \) is specified. Typical grid resolution of \( 150 \times 66 \times 156 \) is used and was verified to be sufficiently converged [12]. The wing is infinitely thin in the limit of \( \Delta x \to 0 \) but, with finite resolution, the wing thickness is about one cell width after discretization due to the use of a regularized delta function (which is the nature of the current immersed boundary method).

The low-aspect-ratio wing is instantaneously materialized at \( t = 0^+ \), corresponding to an impulsive translation. It can also be viewed as having potential flow as the initial condition. In this paper, the temporal variable \( t \) is taken to be nondimensionalized by the free stream velocity \( U_{\infty} \) and the chord \( c \). The time step is chosen such that \( U_{\Delta t}/\Delta x \leq 0.5 \).

This computational setup has been validated against an oil tow-tank experiment at \( Re = 100 \) for a flat rectangular wing of \( AR = 2 \) [12]. Both transient and steady vorticity fields as well as the lift and drag coefficients have been found to be in good agreement for all angles of attack. For further details on the numerical approach and its validation, see [12,25,27,28].

Throughout this work, we consider a Reynolds number of \( Re = U_{\infty}c/v = 300 \), which is above the critical Reynolds number for shedding yet low enough to consider separation in the laminar regime. The lift and drag coefficients are reported as \( C_L = \frac{1}{2} \rho U_{\infty}^2 b c \) and \( C_D = \frac{1}{2} \rho U_{\infty}^2 b c \), respectively, where \( b \) is the wing span. The aspect ratio of the rectangular wing is defined as \( AR = b/c \).

#### Actuator Model

We consider using a model of the ZNMF actuator [29], also known as the synthetic jet, to modify the dynamics of the wake vortices and the corresponding forces exerted on the wing. This actuator has been found to be more effective in enhancing the aerodynamic characteristics of airfoils than steady blowing or suction [18,29]. The current model for the ZNMF actuator is a sinusoidal external force added to the momentum equation:

\[
\mathbf{f}_{\text{act}} = \tilde{f}_{\text{act}} \frac{1 - \cos(2\pi f t)}{2} \hat{z}(x - x_0) \hat{y}(y - y_0) \times \Gamma \left(-z + \frac{b}{2}, z + \frac{b}{2}\right)
\]

where the blowing direction and magnitude is specified with \( \tilde{f}_{\text{act}} \) and the actuation frequency \( f \) is prescribed. This forcing function has maximum and minimum magnitudes of \( \tilde{f}_{\text{act}} \) and 0, respectively, The control force is applied at \( (x_0, y_0) \) through the discrete delta function \( \delta() \) [30] and the Heaviside step function \( \Gamma() \). Here, we have assumed that the momentum added to the fluid during the suction phase is negligible (i.e., fluid sink). In the present model, \( no \) is added or subtracted from the continuity equation. In the case of steady forcing, the term \( (1 - \cos(2\pi ft))/2 \) is replaced by unity.

Below we report the control effort with the steady and oscillatory momentum coefficients

\[
\tilde{C}_D = \rho \tilde{u}_{act} \frac{s b}{\tilde{u}_{act} c b} \quad \text{and} \quad \langle C_D \rangle = \rho \langle \tilde{u}_{act} \rangle \frac{s b}{\tilde{u}_{act} c b}
\]

respectively, where the quantities with overbar and \( \langle \rangle \) represent time-averaged and root-mean-squared values. The slot width \( s \) is set to be the effective width of the regularized delta function [30] that is used to represent the actuator and is \( s/c = \Delta x/c = 0.04 \). To measure the actuator velocity \( u_{act} \), a companion simulation is performed with the same actuation force near the wing in quiescent flow (i.e., \( U_{\infty} = 0 \)). This characteristic velocity is measured as the spatial average of the
resulting steady-state velocity over the support of the regularized delta function along the midspan plane.

In the previous steady actuation study [25], we observed that the most effective controller setup is the downstream blowing (tangential to the wing surface) applied along the entire trailing edge as a strip. Hence, we consider the same control setup in the present investigation. The actuator is placed three grid cells (3Δx/c = 0.12) perpendicularly above the top surface of the wing to avoid numerical interference between the actuator and the immersed boundary force. A schematic of the actuator setup is presented in Fig. 1.

**Poststall Flow Control**

When a wing at a poststall angle of attack undergoes an impulsive translation, the wing experiences high lift due to the creation of the leading-edge vortex and its low-pressure core [12]. The enhancement in lift is lost, however, when this leading-edge vortex detaches, similar to dynamic stall. A large time after the impulsive start, the lift value is at a substantially lower level. The use of actuation is considered to modify the dynamics of the wake in order to increase lift to the level that is achieved early in time. Our objective is not to delay stall or reattach the lift to the level that is achieved early in time. Our objective is not to delay stall or reattach the flow, but to increase lift by modifying the dynamics of the wake vortices behind the low-aspect-ratio wings at poststall angles of attack.

It has been observed that steady downstream injection of momentum along the trailing edge is effective in increasing lift [25]. As the actuator applies steady downstream blowing, the trailing-edge vortex sheet is pushed away from the leading-edge vortices and it rolled into the tip vortices. Such roll-up strengthens the tip vortices and increases the downward-induced velocity by the tip vortices, which in turn repositions the leading-edge vortices closer to the top surface of the wing generating significant lift increase. An illustration of the lift-enhancement mechanism is presented in Fig. 2 (reproduced from [25]). For a momentum coefficient of 0.5% ≤ Cμ ≤ 1.0%, lift enhancement of 22 to 143% is attained for wings with various aspect ratios from 1 to 4, at angles of attack up to 40°.

**Application of Periodic Excitation**

Without control, flow around a low-aspect-ratio wing at large time can exhibit a steady state, a periodic shedding, or aperiodic shedding depending on the aspect ratio of the wing and the angle of attack. When the flow is periodic, the dominant features are the shedding of the leading- and trailing-edge vortices at a constant frequency of

\[
St = f_s c \sin \alpha / U_\infty \approx 0.12
\]  

(6)

as discussed in [12]. This relation will be extended for angles of attack outside of the periodic states in actuating the flow with blowing. In what follows, we report the actuation frequency as nondimensionalized by the natural shedding frequency, i.e., \(f/f_n\).

For details on the shedding behavior and the stability of the wake, see [12].

We first consider the application of periodic forcing around a wing of \(AR = 2\) at \(\alpha = 30°\). Actuation is introduced along the entire trailing edge and directed downstream, parallel to the wing surface. While the current case results in aperiodic flow without control, the excitation frequency is selected by extrapolating the natural shedding frequency for the periodic-shedding case [Eq. (6)], i.e., \(f_s = 0.12U_\infty/(c \sin \alpha)\).

Results with periodic forcing of \(C_e = 0.5\%\) and \((C_m) = 0.25\%\) at \(f/f_s = 1\) are presented in Fig. 3. Shown are the traces of lift and drag coefficients in comparison with cases without control and with steady blowing at \(C_e = 0.5\%\) and 1.0%. These values of the momentum coefficients are based on [25] such that they are at the lowest level resolvable yet exhibiting observable changes in the lift and drag behavior. Here, the maximum forcing magnitude for the periodic forcing case is set to the magnitude of the force in the steady case. From the figure, it can be observed that significant lift increase is similarly achieved with oscillatory actuation as in the steady blowing case. Even without constantly engaging the actuation, we are able to attain lift enhancement at the level between those that are attained by steady momentum injection with \(C_e = 0.5\%\) and 1.0%. We note in passing that with control drag is not increased as much as the lift coefficient in comparison to the unactuated case.

The corresponding wake structures are visualized in Fig. 4 to explain why periodic excitation achieves lift enhancement similar to the level reached by steady forcing. Shown are the isosurfaces of \(\|\omega\| = 3\) and \(Q = 2.5\) (Q criterion [31]), used to capture the vortices and their corresponding cores, respectively. While there are no distinct structures of the tip vortices without actuation, we observe that strong tip vortices are formed with both steady and periodic control. However, the tip vortices with periodic forcing exhibit variation in the strength of the vortex cores in the streamwise direction. By not constantly blowing, the trailing-edge vortex sheet is not always rolled up in the tip vortices. This generates a spanwise variation in the tip vortices. Nonetheless, those tip vortices maintain the self-induced velocity around themselves which presses the leading-edge vortices onto the top surface of the wing for lift enhancement. In fact, the vortical structures very close to the wing are almost identical.

We further examine whether periodic excitation is effective for wings of other AR at different angles of attack. Shown in Fig. 5 are the time-averaged lift and lift-to-drag ratio at large times. Also shown for comparison are the cases with steady blowing using \(C_e = 0.5\%\) and 1.0%. The periodic excitation outperforms the steady blowing for the same time-averaged forcing effort \((\tilde{C}_e = 0.5\%)\) in all cases. As in the steadily forced cases, our three-dimensional argument for lift enhancement performs well for various low-aspect-ratio wings. Note that the current control setup effectively enhances the aerodynamic performance for a wide range of poststall angles of attack even at 40°.

**Effect of Actuation Frequency**

Next, the actuation frequency is varied from 0.1 to 10 times the natural shedding frequency \(f_s\) around an AR = 2 wing at \(\alpha = 30°\). The forcing inputs of \(C_e = 0.5\%\) and \((C_m) = 0.25\%\) are selected as in the previous examples. In Fig. 6, the variations in time-averaged lift and lift-to-drag ratio are shown for different forcing frequencies. The shades represent the amplitude of oscillation in the lift coefficient. Also illustrated by the horizontal lines are the values for the uncontrolled case. Both lift and lift-to-drag ratio exhibit similar response to the change in actuation frequency. Because the lift response shows larger variation in magnitude over the range of actuation frequencies (since there is some increase in drag also as lift is enhanced), the following discussions will focus on the behavior of lift.
For flows excited with frequency above \( f_n \), high-frequency modulation generated by the forcing frequency is observed. Shown in Fig. 7a is the case in which \( f/f_n = 4 \). The overall trends in terms of the average, minimum, and maximum of the lift are similar to the case in which the blowing frequency is \( f_n \).

One interesting case is observed when we choose to blow the trailing-edge vortex sheet at a frequency slightly less than \( f_n \) as shown in Fig. 7b. For \( f/f_n = 0.75 \), the minimum value of lift is also enhanced, shifting the time-averaged lift and lift-to-drag ratio to the largest values out of all frequencies considered here. This agrees with the findings of Seifert et al. [18], which note that the optimal behavior occurs when \( F^+ = fL/U_{\infty} = O(1) \), where \( L \) is the characteristic length of the separated region (in our case \( F^+ \approx 0.12/\sin \alpha \approx 0.12/\alpha = O(1) \)). As we examine the lift trace, we notice that the there is regular shedding. The difference between this case with other cases is that the roll-up of trailing-edge vortex sheet into the tip vortices (strengthening mechanism) and the formation of the leading-edge vortices are in synchronization.

Shown in Fig. 8 are the snapshots of the corresponding wake, sampled at three equally spaced phases (\( \phi = 0, 2\pi/3, \) and \( 4\pi/3 \)) over...
is selected to be at \(10/0.136\) wing at \(10/0.136\) locks on to a \(\leq\) with ctrl \(f = f\). The flow locks on to a high-lift state by keeping the low-pressure core of the leading-edge vortices as close as possible to the top surface in between their detachment. Note that the actuation and the roll-up of the leading-edge vortices are slightly out of phase, since there is a time delay required for the trailing-edge vortex sheet to be rolled into the tip vortices and for the leading-edge vortex to grow in size.

For forcing frequency of \(f/f_n \leq 0.5\), we observe significant variation in lift over time with a decrease in the minimum lift close to the level of the unactuated case. See for example, Fig. 7c, where \(f/f_n = 0.1\). The tip vortices convect away from the wing in between the occurrence of blowing, resulting in the loss of the tip vortices' columnar structures and their corresponding downward-induced velocity. The key here is to avoid decrease in lift or its large fluctuation over time, by actuating with a time scale less than the time required for the tip vortices to lose their structures.

While actuation at such low frequency may not be desirable for steady operation, we can view this numerical simulation as a case of applying a burst of blowing on a required basis. It is interesting to note that in this case, lift can be greatly increased from the unactuated level. Although the tip vortices need to be reformed, the unsteady mechanism to achieve a boost in lift smoothly over a short duration of time may be of interest for certain applications, such as perching maneuvers where the course of action can be planned a priori.

We can also regard the present control setup to be robust against perturbations in the flow if one is to consider the actuation frequency as a perturbation in the controller. Thus, the use of this control setup with feedback could possibly stabilize the perturbed flowfield in the vicinity of the wing to yield large lift in a robust manner. In the work by Ahuja and Rowley [32], a model-based feedback control is found to stabilize the flow about the unstable steady state for analogous two-dimensional flows around a flat wing. A two-dimensional feedback control study by Joe et al. [33] also realized lift increase in a robust manner.

Recently, work on stabilizing the high-lift state using feedback control designed with the optimal seeking algorithm shows promising results for the same three-dimensional separated flow. For forcing frequency \(f/f_n \approx 0.75\). Preliminary results indicate that lock-on to a similar frequency \(f/f_n \approx 0.8\) achieving lift enhancement is possible. Future work will focus on the design of feedback control law for the enhancement of aerodynamic performance (e.g., increasing \(C_L\) and \(C_L/C_D\), while decreasing \(C_D\)) and the development of control guidelines for various-aspect-ratio wings in different operating conditions.

**Conclusions**

In a continuing effort to explore techniques for enhancing lift on low-aspect-ratio wings in low-Reynolds-number poststall flows, we have considered the application of periodic excitation along the trailing edge. Periodic actuation has been found to be effective in enhancing lift by repositioning the leading-edge vortices closer to the top surface of the wing. This benefit has been achieved by

![Figures](https://example.com/filenames.png)
strengthening the tip vortices and the corresponding downwash-induced velocity exerted on the leading-edge vortices. In comparison to the steady forced cases, the periodically actuated cases have shorter tip-vortex cores. Nonetheless, these vortices are able to sustain self-induced roll-up of the trailing-edge vortex sheet. The current control setup with periodic excitation is found to be effective in enhancing lift and lift-to-drag ratio for various low-aspect-ratio wings and poststall angles of attack. In fact, for the use of the same time-averaged momentum coefficient, the periodic forcing outperforms the steady blowing case. It is also observed that there exists an actuation frequency at which the flow locks on to a high-lift state. Such flow exhibits the synchronization of the growth of the leading-edge vortices and the roll-up of the trailing-edge vortex sheet into the tip vortices.

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