Feedback control is used to suppress oscillations in the subsonic flow past a rectangular cavity. A heuristic feedback law is introduced into 2D direct numerical simulations, using zero-net-mass forcing at the leading edge of the cavity and a pressure sensor at the wall, and the oscillations are reduced by $-13$ dB. Reduced-order models are obtained from full-order direct numerical simulations, and used to design observers to reconstruct the full flow information from a single pressure measurement at the wall, and these observers are shown to be more effective than Linear Stochastic Estimation (LSE), which is commonly used for this purpose. Initial results of a new experiment are also given, and a model for designing zero-net-mass actuators with a desired bandwidth is presented, along with experimental data which supports the model predictions.

I. Introduction

At low supersonic and high subsonic flight Mach numbers typical of tactical aircraft, large amplitude acoustic tones can develop inside aircraft cavities. In the weapons bay these tones (known as Rossiter modes) can lead to early structural fatigue problems. With interest expanding in the area of smaller, lighter and smarter ordnance, the large amplitude acoustic tones raise issues maintaining the integrity of smart ordnance. Acoustic resonance also affects aircraft performance. Not only is the acoustic field enhanced by the resonance, but the mean flow field is also modified. Measurements by McGregor & White$^1$ showed a $250\%$ increase in cavity drag during resonant tone conditions. Clearly, it is desirable to control or suppress the Rossiter modes over a wide range of flight conditions.

Current practice is to deploy leading edge fences when weapons bay doors are opened. These passive control devices thicken the cavity shear layer to reduce tone amplitude. However, an additional drag penalty is paid for the fence, and noise suppression at off-design conditions is not guaranteed. If the tone-producing resonance mechanism can be interrupted without using a fence, then it may be possible to reduce the noise and drag without paying an additional drag penalty. Laboratory scale experiments are now exploring a variety of active control scenarios. An extensive review of active control techniques for cavity tone suppression can be found in Cattafesta, et al.$^2$ A few examples are open-loop forcing using high frequency excitation (a.k.a., Hifex technique) investigated by Stanek,$^3$ and a wide range of techniques examined by Shaw,$^4$ including steady blowing, pulsed-blowing, and high-frequency tone generators. Shaw & Northcraft$^5$ also used a feedback signal to modulate the amplitude of a pulsed jet. Large amplitude blowing was investigated in a flight test with an F-111 by Shaw.$^4$

Closed-loop control, in principle, offers several advantages over open-loop methods of control. A closed-loop architecture can adapt to changing flight conditions. Furthermore, there are fundamental system-theoretic reasons why closed-loop control requires far less power than open-loop control. In particular, open-loop forcing can stabilize an unstable equilibrium point only through nonlinear effects (i.e., finite amplitude is required), while closed-loop forcing can linearly stabilize (i.e., with arbitrarily small amplitude). The low power of closed-loop architectures has also been noted in experiments: for example, Cattafesta, et al.$^6$ used piezoelectric actuators for feedback control on the time-scale of the period of the acoustic wave
with an order of magnitude less power than the open-loop control. However, the added complexity of a closed-loop control system and serious limitations in actuator bandwidth have so far prevented closed-loop control from transitioning to flight vehicle applications. From the fluid dynamics viewpoint, we believe that improvements in understanding of the physics associated with the cavity resonance mechanism will lead to better closed-loop architectures for suppressing the tones.

Both open-loop and closed-loop control experiments are underway at the Princeton Gas Dynamics Laboratory at high subsonic Mach numbers. The experiments aim to extend earlier work done at the U.S. Air Force Academy, which used a model-based control algorithm. Refinements in the plant model and control algorithm used to suppress tones allows better measurements of the system time delays, and improved estimates of the controller performance limitations.

In this paper, we present recent results of simulations using closed-loop control to suppress cavity oscillations, as well as some initial results from an experiment being conducted in parallel with the simulations. The main results of the paper are divided into three sections. In Section II, we discuss a simple, heuristic control law that is shown to reduce oscillations in the cavity simulations. In Section III, we design a dynamic observer to reconstruct the entire flow state using only a single pressure measurement at the wall. Previous studies\[7,8,9,10\] have used Linear Stochastic Estimation (LSE) for this purpose, and we show that the dynamic observer has several advantages over static estimators such as LSE: in particular, the need for fewer sensors, and less sensitivity to sensor noise. Finally, in Section IV we discuss initial results from the experiments, including the design of an actuator which will be used for closed-loop experiments.

II. Feedback control in simulations

Direct numerical simulations of the 2D laminar flow over a rectangular cavity have been presented previously, and have been carefully validated and compared with experimental results.\[11\] Here, we introduce actuation into the simulations, in a way that qualitatively mimics the actuation used in the experiment (see Section IV): in particular, we introduce zero net mass injection, but nonzero momentum imparted to the shear layer. In the simulations, we add a body-force term to the right-hand side of the momentum equation, in the regions shown in Fig. 1. The one-actuator case is designed to represent the actuator used in the experiment in Sec. IV, in which forcing is introduced at the upstream corner of the cavity. The four-actuator case is less physically realistic, but enables better suppression with the heuristic control law used here.

**HEURISTIC CONTROL LAW.** The control we use is as follows: we impose a vertical body force proportional to the vertical velocity at the actuation point, and opposite in direction. The idea is to inhibit shear layer oscillations, and thus reduce the amplitude of overall cavity oscillations, and possibly stabilize the flow altogether.

In order to make the single-actuator case as physically realistic as possible, instead of sensing the velocity at the actuation point, we instead sense the pressure at the downstream wall, at \(y = -0.5D\) (see Fig. 1). We then feed back this pressure with a time delay to match the phase difference between vertical velocity of the shear layer at the actuation point, and the pressure at the sensor location, which was determined from previous simulations without control. In order to avoid exciting other frequencies, we first pass the wall pressure signal through a bandpass filter with passband \(St = fL/U \in [0.35, 1.05]\), where \(U\) is the freestream velocity, and \(L\) is the cavity length, as shown in Fig. 1. The frequency of oscillation without control is \(St = 0.7\) (Rossiter mode 2). The performance of the single-actuator case is slightly improved if the
vertical shear layer velocity is sensed directly, but difference is small and these results are not shown. The four-actuator case used collocated sensors and actuators, although of course a correlation with wall pressure is possible here as well.

Figure 2. Pressure at sensor location on downstream cavity wall, with heuristic control law: One actuator (green dashed); four actuators (blue solid); and without control (red dash-dotted).

Pressure traces from the simulations are shown in Fig. 2, for a simulation with aspect ratio $L/D = 2$, Mach number $M = 0.6$, Reynolds number $Re_\theta = 56.8$ based on boundary layer momentum thickness $\theta$ at the cavity leading edge, and $L/\theta = 52.8$. The simulation domain extended to $7.6D$ in the downstream direction, $3.9D$ in the upstream direction, and $9.2D$ in the wall-normal direction. The grid used 1008 $\times$ 384 gridpoints above the cavity and 240 $\times$ 96 gridpoints inside the cavity, which is sufficient to resolve all of the scales at this Reynolds number. The single-actuator case reduces the amplitude of oscillations by 13 dB, and the four-actuator case almost completely eliminates the oscillations, with a reduction of 42 dB.

Although the controller reduces the amplitude of oscillations at the pressure sensor, there is of course no guarantee that the amplitude of oscillations at other points will be reduced, or that the radiated acoustic field will be quieter. Figs. 3 and 4 show instantaneous contours of dilatation and vorticity, respectively, and show that indeed this simple heuristic feedback law has achieved a significant reduction in both the shear layer fluctuations and the radiated acoustic field. (Note that the figures show only a small portion of the full simulation region.)

III. Dynamic estimator

It is often desirable to have information about the entire flow field: for instance, many techniques for control design rely on having full knowledge of the “state,” which in our case is the full flow field in a region of space including the cavity. In practice, one often cannot measure the full flow state directly, and instead one has access only to a limited number of sensors, say pressure sensors at the wall. However, one can often estimate the flow state from the available sensor measurements.

A commonly used method for such prediction is Linear Stochastic Estimation (LSE), introduced by Adrian, which has recently been applied to cavity flows, as well as cylinder wakes and other flows. In this method, one correlates sensor signals with full flow field information from a known database, and then uses the correlation to predict flow field information from the sensor information, when the flow field is not directly available. Higher-order correlations are also possible, and Ukeiley has shown that quadratic stochastic estimation (QSE) outperforms LSE in predicting cavity flow fields.

A disadvantage of the LSE and QSE approaches is that they take into consideration only the sensor measurements at a particular time, and do not use any information about the time history of sensor measurements. Thus, LSE is a “static” estimator, in that the output of the estimator depends only on the input (sensor signals) at that time instant. This has several drawbacks: in particular, the estimator will be very sensitive to noise in the sensor measurement, and also the rank of the estimator (i.e., the dimension of the subspace in which the estimates lie) is at most the number of available sensors. Thus, if one wants to estimate the information in an $n$-dimensional state space, one needs at least $n$ sensors, and preferably more, for a good statistical estimate. For instance, Siegel showed that 10 sensors were needed to reconstruct a reliable estimate of the first two POD modes in a cylinder experiment, to 1–3% accuracy, or the first three
Figure 3. Instantaneous dilatation contours from cavity simulation with heuristic control law. Contour levels are the same for all plots, between \(-0.01\) and 0.01, with a spacing of 0.001 (negative contours dashed).

Figure 4. Instantaneous vorticity contours from cavity simulation with heuristic control law. Contour levels are the same for all plots, between \(-3\) and 3, with a spacing of 0.5 (negative contours dashed).
modes to a 12–20% accuracy.

Here, we consider an alternative approach, a dynamic estimator, which makes use of a reduced-order model of the flow, and uses the time history of the sensor measurements, not just the measurement at a particular time instant. Dynamic estimators or observers are commonly used in the controls community to estimate the state of a system for which a model is available, and these overcome both of the disadvantages of static estimators mentioned above: a single sensor can estimate an arbitrarily high-dimensional state, and optimal observers (Kalman filters) can be designed to trade off the effects of sensor noise and unknown disturbances or modeling uncertainties.

A. Reduced-order model

In order to design a dynamic observer, we require a model of the flow, in the form of a system of ordinary differential equations (ODEs). Here, we will use reduced-order models obtained from data from 2D direct numerical simulations, described in detail in Rowley, Colonius, and Murray. The models are obtained by Galerkin projection of the isentropic Navier-Stokes equations onto POD modes. If \( q \) is a vector of the flow variables of interest, we project the flow field onto a finite set of (spatial) basis functions \( \{ \varphi_j \mid j = 1, \ldots, n \} \), writing

\[
q(x, t) = \bar{q}(x) + \sum_{j=1}^{n} z_j(t) \varphi_j(x),
\]

(1)

where \( \bar{q}(x) \) is some constant flow (typically a steady solution of Navier-Stokes, if known, or in our case a mean flow), and the \( z_j \) are time-varying coefficients. Thus, the state is the vector of coefficients \( z = (z_1, \ldots, z_n) \), and determining the state vector \( z \in \mathbb{R}^n \) specifies the entire flow field \( q \), according to (1). A model is then an evolution equation for \( z(t) \).

Galerkin models are obtained by projecting known dynamics (e.g., the Navier-Stokes equations) onto a smaller-dimensional subspace. Here, we start with the isentropic Navier-Stokes equations, written as

\[
\begin{align*}
\frac{\partial a}{\partial t} &= -v \cdot \nabla a - \frac{\gamma - 1}{2} a \nabla \cdot v, \\
\frac{\partial v}{\partial t} &= -v \cdot \nabla v - \frac{\gamma - 1}{\gamma - 1} a \nabla a + \nu \nabla^2 v,
\end{align*}
\]

(2)

where \( v = (v_1, v_2) \) is the velocity and \( a \) is the local sound speed (which may be related to pressure via isentropic relations). These equations are quadratic in the flow variables, of the form

\[
\dot{q} = L(q) + Q(q, q),
\]

(3)

where \( q = (v_1, v_2, a) \), \( L \) is a linear operator, and \( Q \) is bilinear (linear in each argument).

Using the expansion (1), the model (3) has the form

\[
\dot{z}_i(t) = c_i + A_{ij} \dot{z}_j(t) + Q_{ijk} z_j(t) z_k(t),
\]

(4)

(summation implied), where

\[
\begin{align*}
c_i &= \langle L(\bar{q}) + Q(\bar{q}, \bar{q}), \varphi_i \rangle, \\
A_{ij} &= \langle L(\varphi_j) + Q(\bar{q}, \varphi_j) + Q(\varphi_j, \bar{q}), \varphi_i \rangle, \\
Q_{ijk} &= \langle Q(\varphi_j, \varphi_k), \varphi_i \rangle,
\end{align*}
\]

where we have assumed the basis functions \( \varphi_j \) are orthonormal.

Generically, (4) may have many equilibrium points (e.g., even in one dimension, it may have zero, one, or two equilibria, or a continuum in degenerate cases), but for the cases we investigate, \( \dot{q} \) in (1) is already “close” to an equilibrium point (albeit an unstable one), which will imply that \( c_i \) is small, and (4) has a unique equilibrium point \( z^* \) close to the origin. In developing controllers, we will want to linearize about this equilibrium point, so writing \( z(t) = z^* + \tilde{z}(t) \), one obtains

\[
\dot{\tilde{z}}_i = \tilde{A}_{ij} \tilde{z}_j + Q_{ijk} \tilde{z}_j \tilde{z}_k,
\]

(5)

where \( \tilde{A}_{ij} = A_{ij} + (Q_{ijk} + Q_{ikj}) z^*_k \), so the linearized system is simply \( \dot{\tilde{z}} = \tilde{A} \tilde{z} \), where \( \tilde{A} \) is the \( n \times n \) matrix with components \( \tilde{A}_{ij} \).
B. Observer design

The sensor used in the observer is the same as used in Sec. II for the heuristic control: a wall pressure sensor in the downstream wall of the cavity, at \( y = -0.5D \). This sensor location was not optimized in any way, although one could consider optimal sensor placements by choosing sensor locations where the magnitudes of POD modes are large.\(^9\) Each POD mode \( \varphi_j \) has a corresponding pressure \( p_j \) at this sensor location, and we represent the sensor signal \( \eta(t) \) as

\[
\eta(t) = \sum_{j=1}^{n} \hat{z}_j(t)p_j = C\hat{z}(t)
\]

where \( C \) is the row vector \([p_1 \cdots p_n]\).

We then design a Kalman filter\(^14\) for the linearized system \( \dot{\hat{z}} = A\hat{z} \), where \( A \) is the matrix from (5). Letting \( \hat{z} \) denote the estimate of the actual state \( \hat{z} \), the observer has the form

\[
\dot{\hat{z}} = \hat{A}\hat{z} + L(\eta - C\hat{z})
\]

where \( L \) is a matrix with \( n \) rows and one column (in general, if \( m \) sensors are available, \( L \) has \( m \) columns). The last term in (7) represents corrections proportional to the error in the actual sensor measurement \( \eta \) and the sensor measurement \( C\hat{z} \) predicted by the state estimate \( \hat{z} \). For a Kalman filter, one assumes that there is both sensor noise and process noise (i.e., actual disturbances or modeling uncertainties), both white, Gaussian, zero-mean random variables, and the weights \( L \) are chosen by minimizing the variance of the steady-state error \( \lim_{t \to \infty} E(\hat{z}(t) - \hat{z}(t)) \), for specified variances on the sensor noise and process noise. Here, the process noise variance is estimated from the size of the nonlinear terms in (5). There is very little noise in the pressure measurements in the simulation, but we expect much greater noise in experiments, so we artificially add random noise to the sensor signal, and use this noise variance for designing the Kalman filter gains. Once the observer weights are designed, we consider both the linear observer (7) and the nonlinear observer obtained by adding the correction \( L(\eta - C\hat{z}) \) to the nonlinear system (5).

C. Results and comparison with LSE

The performance of both linear and nonlinear observers is shown in Fig. 5. Here, we used a 4-mode POD/Galerkin model (\( n = 4 \)), presented previously.\(^13\) This model approximates the full simulations well for short times, but for long times, the amplitude of the oscillations is too large. For instance, Fig 5 (right) shows a phase plot of the nonlinear model (orange dotted curve) compared with the projection of the DNS data (black circles), and the amplitude of the model is off by a factor of two. Nevertheless, we use this model in developing the observer, and use the sensor feedback to correct for these modeling uncertainties.

For implementing the observer, a very noisy pressure sensor was used, with noise variance equal to the RMS of the pressure signal, and the signal is shown in Fig. 5 (left). The initial condition was \( \hat{z}(t=0) = 0 \), so the observer was given no information about the initial state of the simulation. Despite the very noisy pressure signal, the observer converges to a very close estimate of the state, within a few oscillation cycles. The linear observer and the nonlinear observer are almost identical for the coefficient of POD mode 1, but the nonlinear observer does much better than the linear observer for POD mode 3. This may suggest that nonlinear coupling between the cavity oscillations at different Rossiter modes is important.

Fig. 5 also compares the performance of the observer with a static estimator obtained by Linear Stochastic Estimation (LSE). Here, three sensors were used, on the front and back cavity walls at \( y = -0.5D \), and on the cavity floor, at \( x = 0.5L \). If noise-free pressure signals are used, the three-sensor LSE produces results very comparable to the one-sensor dynamic observer—in fact, the results are a little better, as there is no transient for LSE, while the observer takes one or two oscillation cycles to lock on to the state. However, the dynamic observer handles the noisy sensor much better than LSE, as seen in Fig. 5. The effect of noise may also be seen in Fig. 6, which shows instantaneous reconstructions of the full flow fields, using noisy pressure sensors. This figure shows how the noisy LSE estimates shown in Fig. 5 manifest themselves as large errors in the overall flowfield prediction.

Furthermore, note that the dynamic observer is able to reconstruct all four states with a single pressure sensor, while the static LSE requires three sensors. (If a one-sensor LSE is used, the phase plot (Fig 5, right) of the state estimate lies along a straight line, instead of a circle, since the rank of the estimate is at most the number of sensor inputs, as discussed earlier.) This is a significant advantage of the dynamic observer,

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Figure 5. Left: Time traces of pressure sensor and POD modes 1 and 3; Right: Phase plot of POD modes 1 and 2. Exact projection of DNS (black ◦), Linear observer using 1 sensor (red dashed); Nonlinear observer using 1 sensor (blue solid); LSE using three sensors (green dashed—shown on left plot only); Nonlinear model without sensor feedback (orange dotted—shown on right plot only).

Figure 6. Instantaneous contours of dilatation, from exact simulation (left), and estimate from dynamic observer (center) and LSE (right), using noisy pressure signals.
as decreasing the number of sensors required decreases the complexity of a feedback control system, and increases the likelihood that closed-loop control will be viable in actual applications.

IV. Model-based actuator design

A cavity for flow control experiments at subsonic Mach numbers recently installed at the Princeton University Gas Dynamics Laboratory is shown in Fig. 7. The facility is capable of achieving freestream Mach numbers in excess of $M = 0.7$. The cavity is 6 in long, 3 in wide and has a variable depth from 0 to 3 in. Following the recommendations of Kegerise and Cattafesta, a “mirror image” cavity was filled with foam above the test cavity to reduce the effects of vertical tunnel modes. The effect of the foam impedance is compared in Fig. 8, where the spectrum is plotted against Mach number and superposed with the first five modes of the Rossiter formula. In Fig. 8 (left) the mirror-image cavity was filled with a dense, closed-cell foam, which acted like a hard surface. Beginning around $M = 0.5$, the Rossiter modes lock on to a specific frequency, which does not change as the Mach number increases. The hard foam was replaced with an open-cell acoustic foam used in anechoic chambers. The resonant frequencies no longer show the lock-on behavior in Fig. 8 (right), and good agreement with the empirical Rossiter formula is seen.

![Figure 7. Photograph of cavity model and actuator.](image)

![Figure 8. Left: Hard foam Rossiter plot result; Right: soft foam Rossiter plot result.](image)

Actuation for the flow control experiments is done with a Selenium D405Ti compression driver (similar to the driver used by Samimy & DeBiasi at Ohio State University,\textsuperscript{15}) consisting of a 200 W voice coil enclosed...
in a pressure tight housing. A 900 W Fender amplifier powered the actuator.

Much of actuator design for flow control is by trial-and-error, and little predictive capability exists to guide the design. Some notable exceptions are the use of lumped element models by Gallas, et al., Rathnasingham & Breuer, and McCormick when the size of the actuator is much smaller than the acoustic wavelength. However, the acoustic driver used in this experiment is large enough that a transition duct is needed to connect it with the cavity; hence, the lumped element approximation is not useful. Instead, we designed the transition duct to be an exponential horn between the compression driver and the actuator exit. Exponential horns have an acoustic impedance and acoustic power that is constant with distance through the horn. Perhaps more important is the existence of a simple theory for predicting the velocity and pressure fluctuation output levels, which can be used to guide the design.

![Figure 9. Pressure amplitude results from actuator (blue dashed) compared with horn theory (red solid).](image)

Using Webster’s horn equation and an exponential area variation, the horn shape can be chosen to design a specific low frequency cutoff, $f_c = mc_0/(4\pi)$, where the flare constant $m$ is the rate of exponential area variation (see below), and $c_0$ is the speed of sound at the entrance to the horn. Frequencies lower than the cutoff value $f_c$ are not passed through the horn. In our design the area decreased from $S_o = \pi \text{ in}^2$ at the driver to a rectangular exit area $S = 0.187 \text{ in}^2$, according to $S(x) = S_o \exp(mx)$, giving a negative flare constant $m = -0.188 \text{ in}^{-1}$, and a predicted cutoff frequency of 200 Hz. Bench tests of the actuator were conducted by frequency sweeps from 0 Hz to 6 kHz input signals. An Endevco pressure transducer and hot-wire anemometer were placed in the exit plane of the actuator slot to record the fluctuating pressures and velocities. The results are shown in Fig. 9. The experimentally measured cutoff frequency was found to be 415 Hz, which is a factor of two larger than predicted. The discrepancy may be related to a Stokes layer effect, which reduces the effective area of the horn near the exit. These types of viscous effects are not accounted for in the theory. The amplitude modulations seen in Fig. 9 are the result of the transmission tube modes. Nevertheless, the overall response of the actuator is quite strong over the range of frequencies required.

The velocity measurements obtained with a hot-wire anemometer indicated peak velocities of 35 m/s with a 2 Vpp input signal. Frequency sweeps from 20 Hz to 10 kHz produced the transfer function between velocity at the actuator exit and the input voltage. The magnitude and phase are shown in Fig. 10. There is a peak in the transfer function at 88 Hz that is not predicted by the horn model, but otherwise the velocity peaks occur at the same frequencies as the pressure peaks.

The effect of open loop forcing on the resonant modes is shown in Fig. 11. The forcing frequency is 1200 Hz, and a harmonic at 2400 Hz can also be seen. This result may be compared with Fig. 8 to see that the open loop forcing reduces resonances beyond $M = 0.55$. Note that scales for spectrogram plots shown in Figs. 8 and 11 are the same. The actuator amplitude is sufficient at this forcing frequency to modify the resonance mechanism, and should be effective in a closed loop control architecture.
Figure 10. Velocity/voltage transfer function for actuator, magnitude and phase.

Figure 11. Spectogram showing effect of 1200 Hz open-loop forcing on the Rossiter plot.
V. Conclusions

We have presented simulations and experiments addressing closed-loop control of oscillations in the oscillating flow past a rectangular cavity. Feedback control was introduced into the simulations, using a heuristic control law, designed to inhibit shear-layer oscillations by imposing a vertical body force at the upstream corner that opposes the vertical motion of the shear layer at the actuation point. The control law was implemented using physically realizable sensors and actuators, and demonstrated a substantial reduction in noise ($-13$ dB).

In addition, a dynamic estimator was used to reconstruct the entire flow-field information from a single pressure sensor. The dynamic estimator has several advantages over static estimators such as LSE: in particular, the dynamic estimator is less sensitive to measurement noise, and fewer sensors are required.

Finally, initial results of an experiment currently underway showed that an actuator design based on Webster’s horn equation predicted the actuator transfer function reasonably well, although for more quantitatively accurate predictions one may need to include boundary layer effects, which were neglected in the current model. The model-based actuator design is particularly important for determining scaling laws for transition from the laboratory to actual aircraft applications.

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